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AN ALGORITHM FOR FREQUENCY-DOMAIN BEAMFORMING OF A DUAL LINE ARRAY

D. R. E. A. TECHNICAL MEMORANDUM 83/F

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Abstract

Left-right ambiguity in towed line array measurements of ambient noise directionality may be prevented by replacing the point sensors with directional elements. Transverse hydrophone pairs from a dual line array can be formed into left- and right- looking cardioid beams for this purpose. Cardioid formation is typically performed in the time domain using analog circuits. This report proposes an alternative approach which operates in the frequency domain. Essentially it is a method of cross-spectral beamforming modified to accomodate cardioid formation. It is readily adaptable to channel gain variation as well as to hydrophone failure. As opposed to the time domain approach, it provides for the use of all functioning hydrophones even when in some pairs one element is lost.

Sommaire

L'ambiguïté transversale (gauche-droite) des mesures de la directivité du bruit ambiant par réseaux en ligne remorqués peut être éliminée en remplaçant les capteurs ponctuels par des éléments directifs. Des paires d'hydrophones transverses d'un réseau en ligne double peuvent servir à cette fin, de façon à produire des faisceaux cardioïdes dirigés vers la gauche ou la droite. La production du faisceau cardioïde s'effectue habituellement dans le domaine temporel au moyen de circuits analogiques. Le présent rapport suggère une autre méthode, qui agit dans le domaine des fréquences. Il s'agit essentiellement d'une méthode de mise en forme du faisceau par spectre croisé, modifiée en vue de la production d'un faisceau cardioïde. L'adaptation de cette méthode aux variations du gain de voie de même qu'aux pannes des hydrophones est immédiate. Contrairement à la méthode dans le domaine temporel, elle permet l'utilisation de tous les hydrophones en état de service, même lorsque l'un des éléments de certaines paires ne fonctionne pas.

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1 Introduction

Horizontal towed line arrays are finding increasing use in mapping the horizontal directionality of undersea acoustic noise. However, such arrays suffer an inherent bearing ambiguity due to their inability to differentiate between signals that arrive from either the left- or right-hand side of the array axis, but that intersect the axis at the same angle. One method of resolving this left-right ambiguity is to use two line arrays separated horizontally by a distance much less than the wavelength of sound at which the individual arrays are designed to operate. This configuration is appropriately termed a dual line array. The bearing ambiguity is removed by synthesizing left- and right-looking cardioid elements using the transverse pairs of hydrophones in the dual line. These cardioid elements are then beamformed using conventional techniques, thereby forming two sets of beams, with each set confined to the half-space on either side of the array.

This report proposes an algorithm for frequency-domain beamforming of the dual line array. It is essentially a cross-spectral beamformer, modified to accomodate cardioid formation. The cross-spectral realization can readily adapt to hydrophone failure and to variations in channel gains across the array. Hence, it is an efficient method to use in processing dual line array data for measurement of ambient noise directionality.

2 Beamforming with well balanced elements

We first consider the situation where all the hydrophones in the array are well-matched in channel gains. The configuration of a dual line array with N port(left) and N starboard(right) omni elements is shown in Figure 1. N port and N starboard-looking cardioid elements can be synthesized by appropriately combining the signals of the transverse pairs of omnidirectional array elements.

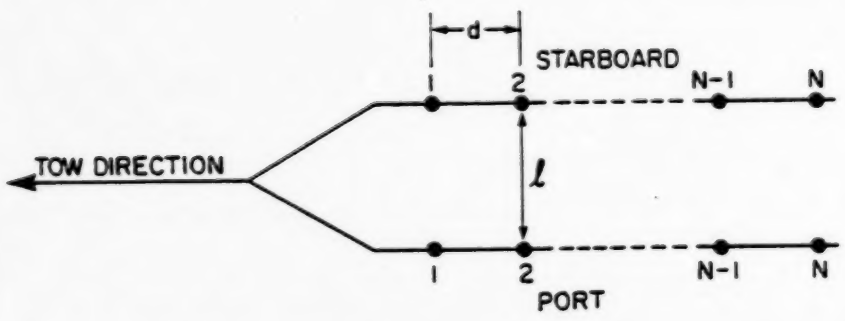

Figure 1. Configuration of a dual line array.

In the time domain, the cardioid signals for the mth transverse pair are:

$$c_m^p(t) = s_m^p(t) - s_m^s(t) + \frac{l\alpha}{2c}\frac{d}{dt}[s_m^p(t)+s_m^s(t)]$$
$$c_m^s(t) = s_m^p(t) - s_m^s(t) - \frac{l\alpha}{2c}\frac{d}{dt}[s_m^p(t)+s_m^s(t)] \qquad (1)$$

where ^{p} and ^{s} indicate port-looking and starboard-looking respectively when applied to the cardioid signals c(t) and indicate port side and starboard side when applied to omni signals s(t),

l is the transverse separation of the elements,

c is the sound velocity,

and α is a constant determining the shape of the cardioids. Cardioid response for small values of kl is $(\alpha-\cos\theta)$, where k is the wavenumber.

Transforming these signals to the frequency domain, and omitting the indication of functional dependence on frequency, we have:

$$C_m^p = S_m^p - S_m^s + j\beta(S_m^p + S_m^s)$$
$$C_m^s = S_m^p - S_m^s - j\beta(S_m^p + S_m^s) \qquad (2)$$

where $\beta = \frac{kl\alpha}{2}$.

The cross-spectrum, C_{km}^{ss}, between the kth and mth starboard-looking cardioids is defined as:

$$C_{km}^{ss} = E(C_k^s C_m^{s\,*}), \qquad (3)$$

where $E(\cdot)$ is the expectation, or time average, of the cardioid signal cross-product. Hence, an estimate, $\hat{C}_{km}^{ss}$, of the cross-spectrum between starboard-looking cardioid elements can be obtained utilizing (2), giving:

$$\begin{aligned}\hat{C}_{km}^{ss} &= C_k^s C_m^{s\,*}\\ &= [(S_k^p - S_k^s) - j\beta(S_k^p + S_k^s)]\,[(S_m^p - S_m^s) - j\beta(S_m^p + S_m^s)]^*\\ &= (1+\beta^2)(\hat{S}_{km}^{pp} + \hat{S}_{km}^{ss}) - (1-\beta^2)(\hat{S}_{km}^{ps} + \hat{S}_{km}^{sp}) + j2\beta(\hat{S}_{km}^{ps} - \hat{S}_{km}^{sp}),\end{aligned} \qquad (4)$$

where $\hat{S}^{ps}_{km}$ is an estimate of the cross-spectrum, S^{ps}_{km}, between the kth element in the port array and the mth element in the starboard array. Likewise for port-looking cardioids:

$$\hat{C}^{pp}_{km} = (1+\beta^2)(\hat{S}^{pp}_{km} + \hat{S}^{ss}_{km}) - (1-\beta^2)(\hat{S}^{ps}_{km} + \hat{S}^{sp}_{km}) - j2\beta(\hat{S}^{ps}_{km} - \hat{S}^{sp}_{km}) . \qquad (5)$$

Thus the cross-spectra between cardioid elements can be expressed in terms of the cross-spectra between single elements.

An estimate of the element cross-spectral matrix, S, is generally obtained by the method of periodograms, whereby a number of consecutive or overlapping time segments of the signals are Fourier transformed, the cross-spectral terms calculated and an average of all of the time samples or "snap shots" taken. It is obvious from examining (4) and (5) that it is not necessary to calculate the cardioid element cross-spectra for each snap shot, but rather the estimated (time-averaged) cardioid element cross-spectra can be determined by operating on the time-averaged element cross-spectra. In the following discussion, we will implicitly assume that the element and cardioid cross-spectra are in fact these time-averaged cross-spectra, and the $\wedge$ will be dropped. We will note where the use of estimated cross-spectra affects any conclusions we draw.

To make this cardioid formation efficient of computer storage requires a reduction in the number of element cross-spectral terms carried through the averaging process. Reductions in the number of terms carried for line arrays of equispaced elements are readily made by taking advantage of the Hermitian property of all cross-spectral matrices and by making the assumption that all terms on a diagonal are equal in the mean; that is the matrix is Toeplitz. In order to have a Toeplitz matrix the acoustic field must be homogeneous, i.e. the signal received from any direction is independent of that received in any other direction. If a Toeplitz assumption is made when the field is non-homogeneous there will be differences between the full matrix and the matrix expanded from the subset of matrix terms carried through time averaging. Given a line array of equispaced elements matched in phase and amplitude, the Toeplitz property suggests the cross-spectral matrix can be reduced to a single row, by combining the terms with redundant element spacings. The resulting row vector is referred to as the spatial correlation vector.

The operation defining the terms of the spatial correlation vector for a line of starboard-looking cardioids is given below:

$$C^s_n = \frac{1}{N-n} \sum_{k=1}^{N-n} C^{s\,s}_{k\,k+n} \qquad n = 0,\cdots\cdots,N-1 \qquad (6)$$

where N is the number of cardioid elements
and n is the index of spatial separation.

The vector C^s_n will be referred to as the spatial correlation vector for starboard-looking cardioid elements. (The expressions for port-looking cardioids are similar.) If there are gain differences between the *cardioid* elements, corrections for such imbalance can readily be applied to the measured cardioid spatial correlation vector defined in (6) to

obtain an estimate of the cardioid spatial correlation vector for a properly balanced array. The estimated spatial correlation vector will be the same, in the mean sense, as the vector for a balanced array, provided the acoustic field is homogeneous.

Once the spatial correlation vectors have been defined they can be Fourier transformed, as discussed in Appendix A, to give line array beam powers in $kd\cdot\cos\theta$ space[1]. The beam responses will be unambiguous in azimuth and either starboard or port-looking depending, of course, on which correlation vector was transformed.

3 The spatial correlation vector, M, of omnidirectional elements

In the previous section the correlation vector was defined in terms of the cross-spectra between cardioid elements (6). As the dual line output consists not of signals from cardioid elements, but rather of signals from omnidirectional elements, it is much more useful to express the cardioid spatial correlation vectors in terms of cross-spectra between omni elements. To this end, we define an omni spatial correlation vector, M, as a concatenation of three vectors:

$$\mathbf{M}^{\dagger} = (\mathbf{M}^{o\dagger}\ \mathbf{M}^{x\dagger}\ \mathbf{M}^{-x\dagger}), \qquad (7)$$

where † is vector transpose. Figure 2 illustrates how the elements of the cross-spectral matrix are combined to obtain the terms in the omni spatial correlation vector, in this example for a dual line array with eight elements per line. The boxed terms in the figure define the three vectors of which M is composed as follows:

$\mathbf{M}^{o}$: The terms of this vector are the averages of the cross-powers between coaxial elements, where the average is across cross-powers for sensors having the same inter-element separation. For an array with N elements per line, this vector will contain N terms.

$\mathbf{M}^{x}$: The terms of this vector are the averages of cross-spectra between pairs of array elements having the same axial separation, but of which one is port and the other starboard, with the starboard element either opposite or aft of the port element. As with $\mathbf{M}^{o}$, the vector contains N terms.

$\mathbf{M}^{-x}$: This vector is similar to $\mathbf{M}^{x}$, except that the port element is now either opposite, or aft, of the starboard element. Note that $M_0^x = M_0^{-x}$.

In total, M contains 3N terms, a significant reduction from the $4N^2$ terms of the complete cross-spectral matrix.

Note that if all the elements are balanced and the acoustic field is homogeneous, the main diagonal elements (element autopowers) of the cross-spectral matrix will be

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[1] d is the sensor separation along the array axis, and θ is the steering direction referenced to the array axis.

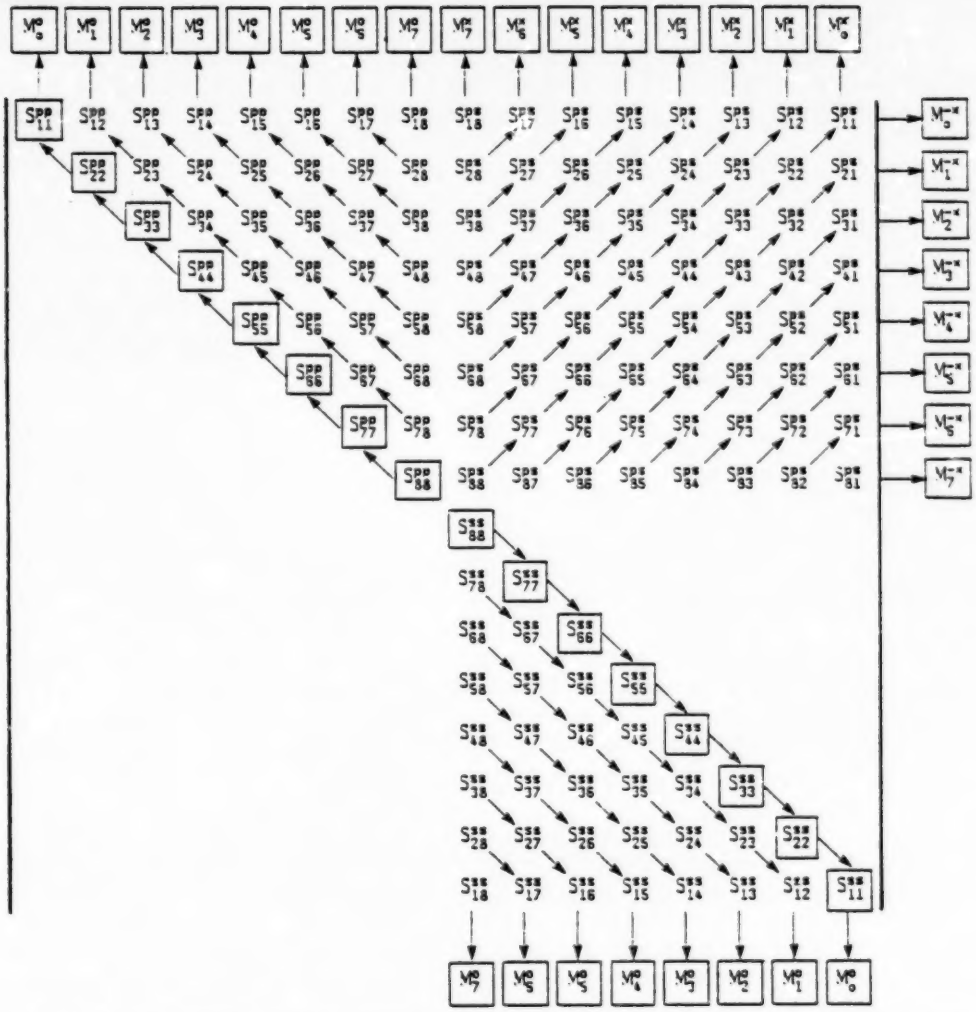

Figure 2. The measured cross-spectral matrix for a 16 element dual line array.

equal in a mean sense, and can be represented by a single term, M_0^o, in the omni spatial correlation vector. However, if channel gains are not equal, the main diagonal elements of the time-averaged cross-spectral matrix contain the information necessary to compensate for this imbalance, and it is important that they be carried throughout. Hence, there is an additional storage of 2N terms required (note that the autopowers have been boxed in Figure 2).

The following algorithm describes the operations performed before time integration of the measured cross-spectra, whereby cross-spectra between array elements with the same relative orientation and spacing are averaged in order to obtain the terms of the omni spatial correlation vector, **M**. If any of the array elements are non-functional due to excessive noise or other causes, the terms in the cross-spectral matrix including these elements should be zeroed before application of the algorithm. Note that in the limit of infinite time averaging, this procedure will lead to a map of the field directionality as measured by the array containing zeroed elements which is the same as that measured by the full array, provided no channel imbalance exists. However, with only a finite time sample available, the estimate obtained from the array

with zeroed elements will be less stable, and in fact, negative estimates of beam power are possible[1].

$$M_n^o = \sum_{k=1}^{N-n} (S_{k\,k+n}^{p\,p} + S_{k\,k+n}^{s\,s}) \Big/ \sum_{k=1}^{N-n} (q_k^p q_{k+n}^p + q_k^s q_{k+n}^s) \qquad n = 0,\cdots\cdots,N-1 \quad (8a)$$

$$M_n^x = \sum_{k=1}^{N-n} S_{k\,k+n}^{p\,s} \Big/ \sum_{k=1}^{N-n} q_k^p q_{k+n}^s \qquad n = 0,\cdots\cdots,N-1 \quad (8b)$$

$$M_n^{-x} = \sum_{k=1}^{N-n} S_{k+n\,k}^{p\,s} \Big/ \sum_{k=1}^{N-n} q_{k+n}^p q_k^s \qquad n = 0,\cdots\cdots,N-1 \quad (8c)$$

where n is the index of spatial separation along the array axis.

The q values are defined as 1 when the element is functional and 0 when the element has been defined as non-functional and the transforms for the element have been zeroed. The sums in the denominator represent the number of occurences of spacing n between functional elements in the array, and hence provide the normalization of the summed cross-spectra to average values[2].

The formation of the omni spatial correlation vector, **M**, for a 4 element dual line array is illustrated in Figure 3. Cross-spectra are calculated between the pairs of elements and contribute to the indicated terms of **M**.

4 Compensation for variations in channel gains

Compensation for variations in the channel gains of array elements can be made at this stage and an omni spatial correlation vector, **B**, can be derived for an array of balanced elements. Channel gain includes both the element sensitivity and any electrical gain following the element. For the m^{th} element at frequency f_i, the channel gain can be estimated by averaging the power over a number of frequency bins about f_i ($i-\delta$ to $i+\delta$) and taking a ratio between the rms amplitudes of the m^{th} element and the reference element r:

$$a_m^{\,2}(f_i) = \sum_{j=i-\delta}^{i+\delta} S_{m\,m}(f_j) \Big/ \sum_{j=i-\delta}^{i+\delta} S_{r\,r}(f_j) \qquad (9)$$

where r is the array element chosen to be the reference. If an element has been identified as non-functional, "a" for that element is arbitrarily set to 0. Note that the superscripts indicating port and starboard are not explicitly included.

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[2] It will be shown in the next section that this normalization is not necessary once the correction for channel gain imbalance is incorporated.

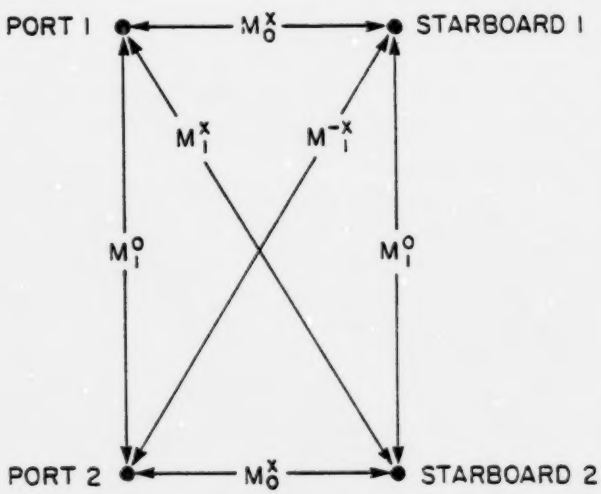

Figure 3. Formation of the omni spatial correlation vector for a dual line array of 4 elements.

Applying the inverse, $1/a_m$, of the channel gains to the appropriate terms in the full cross-spectral matrix will correct for gain imbalances of the elements and alter the matrix to that for an array of balanced elements. The Fourier transform of the signal from the mth element, after it has been balanced with the reference element "r", will be $S_m(f)/a_m(f)$. The diagonal elements of the cross-spectral matrix are altered by application of the gains, so for example S^{pp}_{mm} is replaced by $S^{pp}_{mm}/(a^p_m a^p_m)$ (the indication of functional dependence on frequency has been omitted). Vector **B** can now be obtained by applying the inverse of the channel gains to the cross-spectral terms before summation. The definitions for **B** are the same as those for **M** in (8), except that the cross-spectra under the summation are modified to compensate for this imbalance. Adjusting (8a) thus gives:

$$B^o_n = \sum_{k=1}^{N-n} (S^{p\ p}_{k\ k+n}/a^p_k a^p_{k+n} + S^{s\ s}_{k\ k+n}/a^s_k a^s_{k+n}) \Bigg/ \sum_{k=1}^{N-n} (q^p_k q^p_{k+n} + q^s_k q^s_{k+n}) \quad n=0,\cdots\cdots,N-1 \ . \quad (10)$$

B^o_n, however, cannot be defined in this way unless the entire cross-spectral matrix is available at the end of the time averaging, since estimates of the channel gains are unavailable until this time. Usually, only the omni spatial correlation vector, **M**, for the unbalanced array is retained to reduce computer memory requirements and thus **B** must be obtained by adjusting **M** rather than using the whole cross-spectral matrix, **S**.

For a balanced array the cross-spectral terms for pairs of elements with the

same spacing and orientation in a homogeneous acoustic field will be equal in the mean sense (i.e. the cross-spectral matrix S is Toeplitz). With a Toeplitz matrix the diagonal elements are equal and thus for each spatial separation corresponding to a value of n, each of the terms under summation in the numerator of (10) will equal B_n^o in the mean. Thus:

$$S^p_{k\,k+n} \approx B^o_n \cdot a^p_k\, a^p_{k+n}$$

$$S^s_{k\,k+n} \approx B^o_n \cdot a^s_k\, a^s_{k+n}\,, \qquad k=1,\cdots\cdots,N-n \qquad (11)$$

where the equality is approximate since in practice only estimates of each of the terms in (11) are available. Similarly, for oblique separations:

$$S^p_{k\,k+n} \approx B^x_n \cdot a^p_k\, a^s_{k+n}$$

$$S^p_{k+n\,k} \approx B^{-x}_n \cdot a^p_{k+n}\, a^s_k\,. \qquad k=1,\cdots\cdots,N-n \qquad (12)$$

Using these expressions in conjunction with (8), which defines the vector, M, we can define B in terms of M as:

$$B^o_n = M^o_n \sum_{k=1}^{N-n} (q^p_n q^p_{k+n} + q^s_n q^s_{k+n}) \Big/ \sum_{k=1}^{N-n} (a^p_k a^p_{k+n} + a^s_k a^s_{k+n}) \qquad n=0,\cdots\cdots,N-1$$

$$B^x_n = M^x_n \sum_{k=1}^{N-n} q^p_k q^s_{k+n} \Big/ \sum_{k=1}^{N-n} a^p_k a^s_{k+n} \qquad n=0,\cdots\cdots,N-1 \qquad (13)$$

$$B^{-x}_n = M^{-x}_n \sum_{k=1}^{N-n} q^p_{k+n} q^s_k \Big/ \sum_{k=1}^{N-n} a^p_{k+n} a^s_k \qquad n=0,\cdots\cdots,N-1$$

Note that in (8) the summations over q appear in the denominator, so as to average the cross-spectra sums, while in (13) the identical summations appear in the numerator. Thus it is possible, and expedient, to redefine (8) without this normalization, and likewise remove this term in the definition of the B vector.

5 Cardioid spatial correlation vectors in terms of omni spatial correlation vector

Considering again only starboard-looking cardioids and recalling (4) and (6) we can write:

$$\begin{aligned}\mathcal{C}_n^s &= \frac{1}{N-n}\sum_{k=1}^{N-n} C_{k\,k+n}^{s\,s} \\ &= \frac{1}{N-n}\Bigg[(1+\beta^2)\sum_{k=1}^{N-n}(S_{k\,k+n}^{p\,p}+S_{k\,k+n}^{s\,s}) \\ &\quad -(1-\beta^2)\sum_{k=1}^{N-n}(S_{k\,k+n}^{p\,s}+S_{k\,k+n}^{s\,p}) \\ &\quad +j2\beta\sum_{k=1}^{N-n}(S_{k\,k+n}^{p\,s}-S_{k\,k+n}^{s\,p})\Bigg] \qquad n=0,\cdots\cdots,N-1 \qquad (14)\end{aligned}$$

These equalities were derived under the assumption of balanced elements. To generalize, they can be redefined in terms of the B vector. For a balanced array, B is equivalent to M. Hence, (8) can be reformulated to give:

$$\begin{aligned}&\sum_{k=1}^{N-n}(S_{k\,k+n}^{p\,p}+S_{k\,k+n}^{s\,s}) = 2(N-n)B_n^o \qquad n=0,\cdots\cdots,N-1 \\ &\sum_{k=1}^{N-n}S_{k\,k+n}^{p\,s} = (N-n)B_n^x \qquad n=0,\cdots\cdots,N-1 \qquad (15) \\ &\sum_{k=1}^{N-n}S_{k\,k+n}^{s\,p} = (N-n)B_n^{-x^*} \qquad n=0,\cdots\cdots,N-1\end{aligned}$$

Substituting these expressions into (14), we have:

$$\mathcal{C}_n^s = 2(1+\beta^2)\,B_n^o - (1-\beta^2)\,(B_n^x + B_n^{-x^*}) + 2j\beta\,(B_n^x - B_n^{-x^*}) \qquad n=0,\cdots\cdots,N-1 \qquad (16)$$

Similarly $\mathcal{C}_n^p$ can be defined as:

$$\mathcal{C}_n^p = 2(1+\beta^2)\,B_n^o - (1-\beta^2)\,(B_n^x + B_n^{-x^*}) - 2j\beta\,(B_n^x - B_n^{-x^*}) \qquad n=0,\cdots\cdots,N-1 \qquad (17)$$

With these definitions of the spatial correlation vectors $\mathcal{C}_n^s$ and $\mathcal{C}_n^p$, line array beamforming with port and starboard-looking cardioid elements is completely analogous to beamforming a line array of omnidirectional elements. Cross-spectral beamforming of line arrays is addressed in Appendix A.

6 Acknowledgement

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Appendix A

Cross-spectral beamforming of a line array

Conventional processing of a line array involves time delaying the element signals to be in phase to a plane wave arriving from the steering direction, then amplitude weighting the signals and summing. The time output for such conventional beamforming is:

$$p(t,\theta) = \sum_{m=1}^{N} w_m \, s_m(t-\tau_m) \qquad (1)$$

where $p(t,\theta)$ is the time output signal for a beam steered in direction θ, s_m is the signal from the mth element, and τ_m is the time delay applied to the mth element.

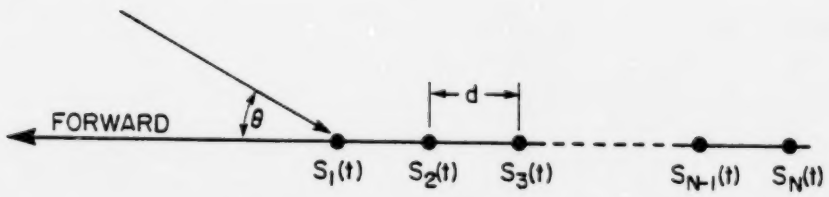

Figure A. Line array geometry.

No assumptions are made here as to the directional characteristics of the elements as this does not affect the beamforming method, only the directional response of the beams. For a line array of equispaced elements as shown in Figure A the time delays required to beamform are:

$$\tau_m = (m-1)d \cos\theta \, / c \, , \qquad (2)$$

where c is the speed of sound and d the inter-element separation. Transforming into the frequency domain and calculating power, we obtain an estimate of the beampower as:

$$|\hat{P}(f,\theta)|^2 = \sum_{k}^{N} \sum_{m}^{N} w_k w_m \, S_k(f) \, S_m(f)^* \exp\left[-j2\pi f \, (\tau_k - \tau_m)\right]$$

$$= \sum_{k}^{N} \sum_{m}^{N} w_k w_m \, S_k(f) S_m(f)^* \exp\left[-jkd \cos\theta \, (k-m)\right] \qquad (3)$$

where k is the wavenumber. Assume the array elements are amplitude balanced and that the array is in a homogeneous acoustic field. In this case the expectation, $|P(\theta)|^2$, of the beampower can be written as:

$$|P(\theta)|^2 = P_0 \sum_{k}^{N} \sum_{m}^{N} w_k w_m \rho_{km} \exp\,[-jkd \cos\theta\,(m-k)]\,, \qquad (4)$$

where P_0 is the omni power and ρ_{km} is the spatial correlation between the k^{th} and m^{th} elements. As we noted earlier, the elements of the array may be directional; in fact in the main body of the report, we consider the case where each element has a cardioid directivity, so that the appropriate spatial correlations are the cardioid spatial correlations. This expression can in turn be reformulated in terms of a new index of summation, n, an index of spatial separation:

$$|P(\theta)|^2 = P_0 \sum_{n=-(N-1)}^{N-1} \rho_n \exp\,(-jnkd \cos\theta) \left(\sum_{k=1}^{N-|n|} w_k\, w_{k+|n|} \right), \qquad (5)$$

where n=m-k and ρ_n is the spatial correlation for elements separated by n spaces. Positive values of n indicate the first element is forward of the second(k<m) and negative values of n indicate the first element is aft of the second(k>m). ρ_{-n} is the complex conjugate of ρ_n. The summation in the round bracket in the above equation is the weight applied to the spatial correlation function ρ_n and can be simplified to W_n. Now defining a parameter l such that:

$$kd \cos\theta = 2\pi\, l\, /N \qquad (6)$$

the expression for beampower can be rewritten as:

$$|P(\theta)|^2 = P_0 \sum_{n=-(N-1)}^{N-1} W_n \rho_n \exp\,[-j2\pi nl/N]\ . \qquad (7)$$

This expression has the form of a discrete Fourier transform. Thus the beampower for a uniformly spaced line array can be determined by performing a discrete Fourier transform on the spatial correlation vector, implemented via the Fast Fourier Transform (FFT) algorithm. This algorithm provides beampowers in N steering directions equally spaced in $kd\cos\theta$ space. In order to obtain beampowers at more than N steering directions the number of points in the transformation has to be increased, which can be done by adding zeros to the correlation matrix before transformation.

References

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13. ABSTRACT

Left-right ambiguity in towed line array measurements of ambient noise directionality may be prevented by replacing the point sensors with directional elements. Transverse hydrophone pairs from a dual line array can be formed into left-and right- looking cardioid beams for this purpose. Cardioid formation is typically performed in the time domain using analog circuits. This report proposes an alternative approach which operates in the frequency domain. Essentially it is a method of cross-spectral beamforming modified to accomodate cardioid formation. It is readily adaptable to channel gain variation as well as to hydrophone failure. As opposed to the time domain approach, it provides for the use of all functioning hydrophones even when in some pairs one element is lost.

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Underwater acoustics

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